12|2|2013

- Subject: Applications of Integration
    - Physics
    - Business
    - Probability
- Sections: 6.6; 6.7; 6.8
- Next: Summary of what do we know well at the end of the semester.

Great Opportunity:

Extra Credit:

- Reflections from the Math 121 Classroom - 3 weeks before the Final Exam : 10 pts One page
- Reports from 3 talks with 3 guest speakers, 5 pts each ½ page
  $3 \times 5 = 15$ pts
- Participating in the evaluation of lecture and labs.
  5 pts

Goals:

- of Students
- of Lab Instructors
- of the Coordinator

Let us think of them.

Thank you !!

Dziękuję !!

Probability - Introduction to Math 526 - Probability and Statistics

- **Probability density function**

  f : $f(x) \geq 0$ for all x

  and $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1.$

Example:

Let $f(x) = \begin{cases} k x^2 (1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x < 0 \text{ or } x > 1 \end{cases}$

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Q: (a) For what value of k is f a probability density function?

(b) For that value of k, find $P(X \geq \frac{1}{2})$

(c) Find the expected value of X.

Solution:

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} k x^2(1-x)\,dx$$

$$= k\int_0^1 (x^2 - x^3)\,dx = k\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= k\left[\left(\frac{1}{3} - \frac{1}{4}\right) - 0\right] = k\,\frac{1}{12}$$

$$\Rightarrow \boxed{1 = k \cdot \frac{1}{12}} \iff \boxed{k = 12}$$

$$\Rightarrow f(x) = \begin{cases} 12x^2(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{if } x < 0 \text{ or } x > 1 \end{cases}$$

is the probability density function.

(b) $$P\left(X \geq \frac{1}{2}\right) = \int_{1/2}^{\infty} 12(x^2 - x^3)\,dx$$

$$= \cancel{\lim_{t\to\infty} \int_{1/2}^{t} 12(x^2 - x^3)\,dx} = \cancel{\lim_{t\to\infty} 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{1/2}^{t}}$$

~~but~~ observe that $f(x) = 0$ for $x > 1$

$$\Rightarrow \int_{1/2}^{\infty} 12(x^2 - x^3)\,dx$$

$$= \int_{1/2}^{1} 12(x^2 - x^3)\,dx$$

$$= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_{1/2}^{1} = 12\left[\frac{1}{12} - \left(\frac{1}{24} - \frac{1}{64}\right)\right]$$

$$= 12\left[\frac{1}{12} - \frac{1}{24} + \frac{1}{64}\right]$$

$$= 1 - \frac{1}{2} + \frac{3}{16}$$

$$= 1 - \frac{8}{16} + \frac{3}{16} = \frac{11}{16}$$

Recall:

$$0 \leq P(A) \leq 1$$

with $P(\emptyset) = 0$

and $P(\Omega) = 1$

$$P(A) = 1 \not\Rightarrow A = \Omega$$

(c) Expected Value:

$$EX := \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^1 12x(x^2 - x^3)\, dx$$

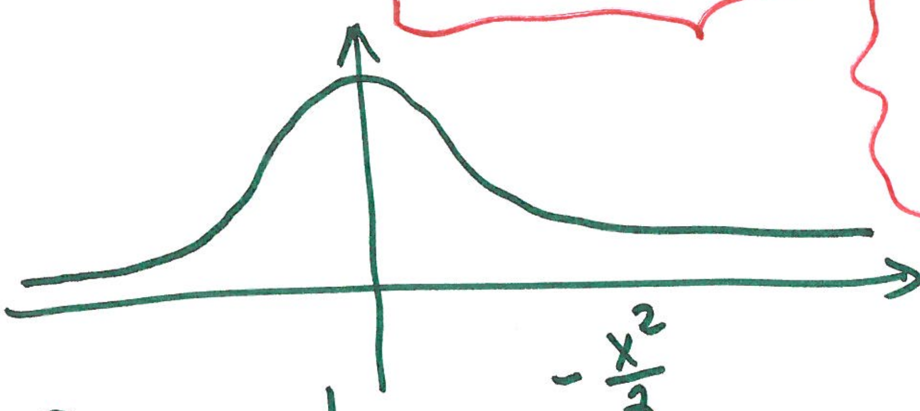
$$= \int_0^1 12(x^3 - x^4)\, dx$$

$$= 12\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 12\left(\frac{1}{4} - \frac{1}{5}\right)$$

$$= 12\,\frac{1}{20} = \frac{6}{10} = 0.6$$

- Standard $\underbrace{\text{Normal Distribution}}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$f(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}} \qquad -\infty < x < \infty$$

$\sigma = 1$ and $\mu = 0$

- Newton's Second Law of Motion:

position $s(t)$

$$F = m\frac{d^2 s}{dt^2}$$

mass in kg

Force in newtons

$N = kg \cdot m/sec^2$

Work done in moving the object from a to b:

$$W = \int_a^b f(x)\,dx$$

force acts on the object

Example:

A particle is moved along the x-axis by the force that measures $10/(1+x)^2$ pounds at a point x feet from the origin. Find the work done in moving the particle from the origin to a distance of 9 ft.

Solution:

$$W = \int_0^9 \frac{10}{(1+x)^2}\,dx = 10\left[-\frac{1}{1+x}\right]_0^9 = 10\left(-\frac{1}{10}+1\right) = \frac{9}{\cancel{10}}$$

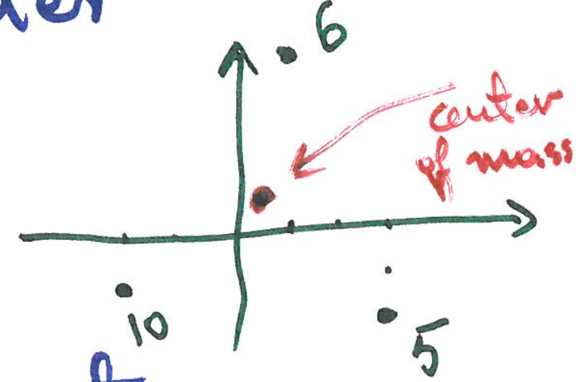
$$\int \frac{1}{(1+x)^2}\,dx = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1}$$

$$= -\frac{1}{1+x}$$

$1 + x = u$

$dx = du$

Moments and center of mass



__Example__:

Masses are located at the points P_i :

$m_1 = 6$, $m_2 = 5$, $m_3 = 10$

$P_1(1, 5)$; $P_2(3, -2)$; $P_3(-2, -1)$

Find the moments M_x and M_y and the center of mass of the system:

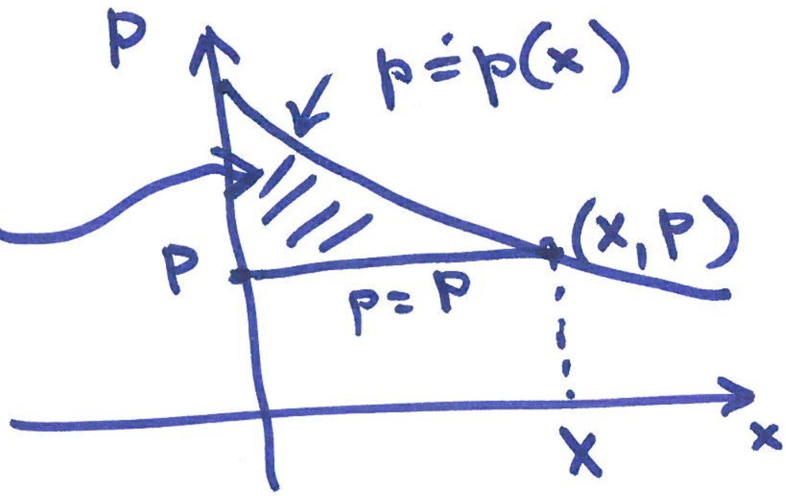
$$M_x = \sum m_i y_i = 6 \cdot 5 + 5 \cdot (-2) + 10(-1) = 10$$

$$M_y = \sum m_i x_i = 6 \cdot 1 + 5 \cdot 3 + 10(-2) = 6 + 15 - 20 = 1$$

The center $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(\frac{1}{21}, \frac{10}{21}\right)$

Consumer Surplus for the commodity

$$\int_0^X [p(x) - P]\, dx$$



The consumer surplus represents the amount of money saved by consumers in purchasing the commodity at price P corresponding to an amount demanded of X.

The figure above shows the interpretation of the consumer surplus as the area under the demand curve and ~~the~~ above the line $p = P$.

Example:

A demand curve is given by
$p = 450/(x+8)$.

Find the consumer surplus when x = 10.
~~when the selling price is $10.~~

Solution:

The consumer surplus is:

$$\int_0^{10} \left[\frac{450}{x+8} - P \right] dx$$

where $P = p(x) = p(10)$

$$= \frac{450}{10+8} = \frac{450}{18} = 45$$

$$= \int_0^{10} \left[\frac{450}{x+8} - 45 \right] dx = \Big[450 \ln(x+8) - 45x \Big]_0^{10}$$

$$= 450(\ln 18) - 450 - 450 \ln 8$$

$$= 450 \left[\ln 18 - \ln 8 - 1 \right]$$

$$= \$450 \left[\ln 2 + 2 \ln 3 - 3 \ln 2 - 1 \right]$$

Review:

- Sec. 6.6 :

  Examples: 1, 6

- Sec. 6.7: 1

- Sec. 6.8 : 1, 3, 4.

Q: Which topic from the course is your favorite one? Why?

Essay Question. Think of it.

Q: Which application of calculus is the most fascinating for you? Think of it.