Math 728 Workshop I | 02|05|2013

1. Let $X_1, X_2, \ldots$ be independent Bernoulli random variables, $X_i \sim \mathrm{BIN}(1, p_i)$, and let $Y_n = \sum_{i=1}^{n} (X_i - p_i)/n$. Show that the sequence $Y_1, Y_2, \ldots$ converges stochastically to $c = 0$ as $n \to \infty$. *Hint:* Use the Chebychev inequality.

2. Suppose that $Z_i \sim \mathrm{N}(0, 1)$ and that $Z_1, Z_2, \ldots$ are independent. Use moment generating functions to find the limiting distribution of $\sum_{i=1}^{n} (Z_i + 1/n)/\sqrt{n}$.

3. Consider a random sample from a Poisson distribution, $X_i \sim \mathrm{POI}(\mu)$.
   (a) Show that $Y_n = e^{-\bar{X}_n}$ converges stochastically to $P[X = 0] = e^{-\mu}$.
   (b) Find the asymptotic normal distribution of Y_n.
   (c) Show that $\bar{X}_n \exp(-\bar{X}_n)$ converges stochastically to $P[X = 1] = \mu e^{-\mu}$.

4. Find the MLE for θ based on a random sample of size n from a distribution with pdf

$$f(x; \theta) = \begin{cases} 2\theta^2 x^{-3} & \theta \leqslant x \\ 0 & x < \theta; 0 < \theta \end{cases}$$

5. Let $X \sim \mathrm{POI}(\mu)$, and let $\theta = P[X = 0] = e^{-\mu}$.
   (a) Is $\hat{\theta} = e^{-X}$ an unbiased estimator of θ?
   (b) Show that $\tilde{\theta} = u(X)$ is an unbiased estimator of θ, where $u(0) = 1$ and $u(x) = 0$ if $x = 1, 2, \ldots$
   (c) Compare the MSEs of $\hat{\theta}$ and $\tilde{\theta}$ for estimating $\theta = e^{-\mu}$ when $\mu = 1$ and $\mu = 2$.