HW #1:

4.1.2. The weights of 26 professional baseball pitchers are given below; [see page 76 of Hettmansperger and McKean (2011) for the complete data set]. Suppose we assume that the weight of a professional baseball pitcher is normally distributed with mean μ and variance σ^2.

160	175	180	185	185	185	190	190	195	195	195	200	200
200	200	205	205	210	210	218	219	220	222	225	225	232

(a) Obtain a frequency distribution and a histogram or a stem-leaf plot of the data. Use 5-pound intervals. Based on this plot, is a normal probability model credible?

(b) Obtain the maximum likelihood estimates of μ, σ^2, σ, and μ/σ. Locate your estimate of μ on your plot in part (a).

(c) Using the binomial model, obtain the maximum likelihood estimate of the proportion p of professional baseball pitchers who weigh over 215 pounds.

(d) Determine the mle of p assuming that the weight of a professional baseball player follows the normal probability model $N(\mu, \sigma^2)$ with μ and σ unknown.

4.1.6. Show that the estimate of the pmf in expression (4.1.9) is an unbiased estimate. Find the variance of the estimator also.

4.4.5. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size 4 from the distribution having pdf $f(x) = e^{-x}$, $0 < x < \infty$, zero elsewhere. Find $P(Y_4 \geq 3)$.

4.4.9. Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample of size n from a distribution with pdf $f(x) = 1$, $0 < x < 1$, zero elsewhere. Show that the kth order statistic Y_k has a beta pdf with parameters $\alpha = k$ and $\beta = n - k + 1$.

5.1.2. Let the random variable Y_n have a distribution that is $b(n, p)$.

(a) Prove that Y_n/n converges in probability to p. This result is one form of the weak law of large numbers.

(b) Prove that $1 - Y_n/n$ converges in probability to $1 - p$.

(c) Prove that $(Y_n/n)(1 - Y_n/n)$ converges in probability to $p(1 - p)$.

5.1.3. Let W_n denote a random variable with mean μ and variance b/n^p, where $p > 0$, μ, and b are constants (not functions of n). Prove that W_n converges in probability to μ.
Hint: Use Chebyshev's inequality.

5.2.1. Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.