HW #2

4.2.1. Let the observed value of the mean $\overline{X}$ and of the sample variance of a random sample of size 20 from a distribution that is $N(\mu, \sigma^2)$ be 81.2 and 26.5, respectively. Find respectively 90%, 95% and 99% confidence intervals for μ. Note how the lengths of the confidence intervals increase as the confidence increases.

4.2.12. Let Y be $b(300, p)$. If the observed value of Y is $y = 75$, find an approximate 90% confidence interval for p.

4.2.17. It is known that a random variable X has a Poisson distribution with parameter μ. A sample of 200 observations from this distribution has a mean equal to 3.4. Construct an approximate 90% confidence interval for μ.

4.2.18. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, where both parameters μ and σ^2 are unknown. A *confidence interval* for σ^2 can be found as follows. We know that $(n-1)S^2/\sigma^2$ is a random variable with a $\chi^2(n-1)$ distribution. Thus we can find constants a and b so that $P((n-1)S^2/\sigma^2 < b) = 0.975$ and $P(a < (n-1)S^2/\sigma^2 < b) = 0.95$.

(a) Show that this second probability statement can be written as

$$P((n-1)S^2/b < \sigma^2 < (n-1)S^2/a) = 0.95.$$

(b) If $n = 9$ and $s^2 = 7.93$, find a 95% confidence interval for σ^2.

(c) If μ is known, how would you modify the preceding procedure for finding a confidence interval for σ^2?

4.2.21. Let two independent random samples, each of size 10, from two normal distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ yield $\overline{x} = 4.8$, $s_1^2 = 8.64$, $\overline{y} = 5.6$, $s_2^2 = 7.88$. Find a 95% confidence interval for $\mu_1 - \mu_2$.

4.2.22. Let two independent random variables, Y_1 and Y_2, with binomial distributions that have parameters $n_1 = n_2 = 100$, p_1, and p_2, respectively, be observed to be equal to $y_1 = 50$ and $y_2 = 40$. Determine an approximate 90% confidence interval for $p_1 - p_2$.

4.2.27. Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be two independent random samples from the respective normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, where the four parameters are unknown. To construct a *confidence interval for the ratio*, σ_1^2/σ_2^2, of the variances, form the quotient of the two independent χ^2 variables, each divided by its degrees of freedom, namely,

$$F = \frac{\frac{(m-1)S_2^2}{\sigma_2^2}/(m-1)}{\frac{(n-1)S_1^2}{\sigma_1^2}/(n-1)} = \frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2},$$

where S_1^2 and S_2^2 are the respective sample variances.

(a) What kind of distribution does F have?

(b) From the appropriate table, a and b can be found so that $P(F < b) = 0.975$ and $P(a < F < b) = 0.95$.

(c) Rewrite the second probability statement as

$$P\left[a\frac{S_1^2}{S_2^2} < \frac{\sigma_1^2}{\sigma_2^2} < b\frac{S_1^2}{S_2^2}\right] = 0.95.$$

The observed values, s_1^2 and s_2^2, can be inserted in these inequalities to provide a 95% confidence interval for σ_1^2/σ_2^2.