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**Exam Questions**

1. (Convergence in Probability) Let the random variable Y*n* have a distribution that is *b*(*n*, *p*).

(a) Prove that Y*n* / *n* converges in probability to *p*. This result is one form of the weak law of large numbers.

(b) Prove that 1 – Y*n* / *n* converges in probability to 1 – *p*

(c) Prove that (Y*n* / *n*)(1 – Y*n* / *n*) converges in probability to 1 – *p*.

**Hogg, McKean, & Craig (7th Ed.) #5.1.2**

1. (Maximum Likelihood Estimator) Let X­1,, X2, ..., X­n be a random sample from a *N*(*θ*, *σ*2) distribution, where *σ*2 is fixed but -∞ < *θ* < ∞.

(a) Show that the mle of *θ* is $\overbar{X}$

**Hogg, McKean, & Craig (7th Ed.) #6.1.10a**

1. (Confidence Intervals) Let the observed value of the mean $\overbar{X}$ and of the sample variance of a random sample of size 20 from a distribution that is *N*(*µ*, *σ*2) be 81.2 and 26.5, respectively. Find respectively 90%, 95% and 99% confidence intervals for *µ*. Note how the lengths of the confidence intervals increase as the confidence increases.

**Hogg, McKean, & Craig (7th Ed.) #4.2.1**

1. (Testing Hypotheses + Power Functions) Let us say the life of a tire in miles, say *X*, is normally distributed with mean *θ* and standard deviation 5000. Past experience indicates that *θ* = 30,000. The manufacturer claims that the tires made by a new process have mean *θ* > 30,000. It is possible that *θ* = 35,000. Check his claim by testing *H0* : *θ* = 30,000 against *H1* : *θ* > 30,000. We observed n independent values of *X*, say *x1*, …, *xn*, and we reject *H0* (thus accept *H1*) if and only if $\overbar{x}$ ≥ *c*. Determine *n* and *c* so that the power function γ(*θ*) of the test has the values γ(30,000) = 0.01 and γ(35,000) = 0.98.

**Hogg, McKean, & Craig (7th Ed.) #4.5.8**

1. (Point Estimator + Unbiasedness) Show that the estimate of the pmf in expression (4.1.9) is an unbiased estimate. Find the variance of the estimator also.

$\hat{p}\left(α\_{j}\right)= \frac{1}{n}\sum\_{i=1}^{n}I\_{j}(X\_{i})$, where $I\_{j}\left(X\_{i}\right)=\left(\begin{array}{c}1 X\_{i}=a\_{j}\\0 X\_{i}\ne a\_{j}\end{array}\right.$ (4.1.9)

**Hogg, McKean, & Craig (7th Ed.) #4.5.8**

1. (Chi-square Tests) Consider the problem from genetics of crossing two types of peas. The Mendelian theory states that the probabilities of the classifications (a) round and yellow, (b) wrinkled and yellow, (c) round and green, and (d) wrinkled and green are $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$, respectively. If, from 160 independent observations, the observed frequencies of these respective classifications are 86, 35, 26, and 13, are these data consistent with the Mendelian theory? That is, test, with ɑ = 0.01, the hypothesis that the respective probabilities are $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$.

**Hogg, McKean, & Craig (7th Ed.) #4.7.4**

1. (Wild-card: Maximum Likelihood Estimator) Let X­1,, X2, ..., X­n be a random sample from a Bernoulli distribution with parameter *p*. If *p* is restricted so that we know that 1/2 ≤ *p* ≤ 1, find the mle of this parameter.

**Hogg, McKean, & Craig (7th Ed.) #6.1.9**

1. (Rao-Cramer Efficiency) Prove that $\overbar{X}$, the mean of a random sample of size *n* from a distribution that is *N*(*θ*, *σ*2), -∞ < *θ* < ∞, is, for every known *σ*2 > 0, an efficient estimator of *θ*

**Hogg, McKean, & Craig (7th Ed.) #6.2.1**